Lesson 24
Solve Problems Using the Pythagorean Theorem

Prerequisite: Finding Square Roots

Study the example showing how to solve a problem that involves square roots. Then solve problems 1–7.

Example

Leroy’s backyard is a rectangle that is three times as long as it is wide. What is the perimeter and the area of the yard?

First draw a diagram to help you solve the problem. Let \( a \) be the width of Leroy’s yard.

![Diagram of Leroy's Backyard]

The perimeter is \( a + 3a + a + 3a \), or \( 8a \). The area of a rectangle is \( ℓ \cdot w \), so the area of the backyard is \( a \cdot 3a \), or \( 3a^2 \).

1. Suppose you know the area of the yard is 7,500 square feet. How can you find the value of \( a \)?

   Solution: ________________________________

2. Use your answer to problem 1 to find the value of \( a \).

   Show your work.

3. What is the perimeter of the backyard?

   Solution: ________________________________

Vocabulary

square root a factor of a number that when multiplied by itself results in the number. For example, 9 is a square root of 81 because \( 9 \times 9 = 81 \).
Solve.

4 The area of the floor of a square room is 144 square feet. What is the perimeter of the room? Include a diagram in your solution.

*Show your work.*

Solution: ____________________________

5 Which two numbers in the box are perfect squares? Explain how you know they are perfect squares.

| 20 | 125 | 196 | 500 | 625 |

6 What is the side length of a square whose area is $\frac{1}{64}$ square mile? Explain your answer.

7 Natasha says that if the area of a square is $a$, then the length of a side of the square is $\sqrt{a}$. Do you agree? Explain.
Study the example showing how to solve a problem that involves distance. Then solve problems 1–5.

Example

Mr. Nichols followed the road from the entrance of a park to his campsite. He drove 6 miles south, then 5 miles east, 3 miles south, and finally 7 miles east to the campsite. In all, how far did Mr. Nichols drive? Suppose Mr. Nichols had been able to drive straight from the entrance of the park to the campsite. How could you find that distance?

You can use the diagram to help you solve this problem. Add the individual distances to find that Mr. Nichols drove a total of $6 + 5 + 3 + 7 = 21$ miles.

You can use the Pythagorean Theorem to find the distance that Mr. Nichols would have driven if he could have driven straight from the entrance to the campsite. The distance is the hypotenuse of a right triangle with leg lengths of 9 miles and 12 miles, so you can write and solve the equation $9^2 + 12^2 = c^2$ and solve for $c$.

1. Find the distance if Mr. Nichols had been able to drive straight from the entrance to his campsite. **Show your work.**

   Solution: _____________________________

   _____________________________

2. How much shorter would Mr. Nichols’ drive have been if he had been able to drive straight from the entrance to his campsite? Explain.

   _____________________________

   _____________________________
Solve.

3 You tie a spherical balloon that is 2 feet in diameter to a stake in the ground. The string is 15 feet long. The wind blows and you observe that the top of the balloon is 8 feet over from the stake, as shown in the diagram. What is the height, $b$, of the balloon?

*Show your work.*

Solution: ________________________________

4 In problem 5, the wind blows harder and the top of the balloon is now 15 feet over from the stake. What is the height of the balloon now? Draw a diagram to help you solve the problem and explain your answer.

5 The perimeter of an equilateral triangle is 48 cm. Use estimation to find the height of the triangle to the nearest whole number. Include a sketch in your answer.

*Show your work.*

Solution: ________________________________
The Pythagorean Theorem and Three-Dimensional Figures

Study the example showing how to apply the Pythagorean Theorem. Then solve problems 1–6.

**Example**

The diagram shows a diagonal drawn from point \( A \) to point \( B \) in a rectangular prism. How can you find the length of this diagonal?

You can use what you know about right triangles that are related to the diagonal.

The triangle on the base of the prism with side lengths 6, 8, and \( c \) is a right triangle. The triangle that includes the diagonal from point \( A \) to point \( B \) with side lengths \( c \), 24, and \( d \) is a right triangle.

Use the Pythagorean Theorem to solve for \( c \). Then use \( c \) to solve for \( d \).

1. How do you know that the triangle with side lengths 6, 8, and \( c \) is a right triangle?

2. How do you know that the triangle with side lengths \( c \), 24, and \( d \) is a right triangle?

3. Use the Pythagorean Theorem to write equations to find the values of \( c \) and \( d \). Find the value of \( c \) and then the value of \( d \).
Solve.

4 Between what two whole numbers is the length of the diagonal from $M$ to $N$ in the rectangular prism?

*Show your work.*

Solution: ________________________________

5 Tyler wants to ship a fishing pole that is 7 feet long. He will use a box that is a rectangular prism with a base that is 4 feet by 3 feet. What is the shortest whole-number height that will hold the fishing pole? Explain.

*Show your work.*

Solution: ________________________________

6 Write a real-world problem about a rectangular prism that can be solved by using the Pythagorean Theorem. Then solve your problem.
Solve Problems Using the Pythagorean Theorem

Solve the problems.

1. Elsa is flying a kite. She let out 25 meters of string. The kite is directly above a spot that is 7 meters away. How high above the ground is the kite?
   
   Show your work.
   
   Solution: ____________________________________________

2. The dimensions of a box that is a rectangular prism are 3 inches, 4 inches, and 6 inches. What is the length of the diagonal from point R to point S, to the nearest tenth of an inch?
   
   A  5 inches  
   B  6.7 inches  
   C  7.8 inches  
   D  61 inches  

   Rodney chose B as the correct answer. How did he get that answer?
   
   ________________________________

3. Margo drew a diagram of a square wooden deck, with side length c feet, that she wants to build in her square yard. Explain how to find the perimeter of the deck. Then find the perimeter.

   Show your work.
   
   Solution: ____________________________________________
Solve.

4 Tell whether each statement is True or False about the right triangle shown.

\[ a \quad b \quad c \]

\[ a. \text{ The square of } b \text{ is always equal to the square of } c \text{ minus the square of } a. \]

\[ \square \text{ True} \quad \square \text{ False} \]

\[ b. \text{ The square of } c \text{ equals the square of } b \text{ plus the square of } a. \]

\[ \square \text{ True} \quad \square \text{ False} \]

\[ c. \text{ The value of } c \text{ is equal to the square of the sum of } b \text{ and } c. \]

\[ \square \text{ True} \quad \square \text{ False} \]

5 An isosceles right triangle has a hypotenuse of 12. Between which two whole numbers are the lengths of the legs of the triangle? 

*Show your work.*

Solution: ______________________________

6 The length of the base of a rectangular prism is 3 feet and its width is 2 feet. The length of the diagonal of the prism is 6 feet. What is the height of the prism to the nearest tenth of a foot?

\[ \begin{array}{ll}
A & 3.3 \text{ ft} \\
B & 4.8 \text{ ft} \\
C & 13 \text{ ft} \\
D & 23 \text{ ft}
\end{array} \]

*You can draw a diagram to help you visualize the problem.*